

Possible Enhanced Flux of Glassy Solid Helium in Cylindrical Corrugated Nanopores

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Abstract

By using the glassy (helium-)fluid model and boundary perturbation method, we can obtain the velocity fields (as well as the flow rate; up to the second order) inside the wavy-rough cylindrical nanopores which are of the same size as those samples prepared by Kim and Chan as well as Day *et al.* Our results show that the velocities measured in porous Vycor samples could be reproduced by carefully selecting relevant parameters but those in glass capillaries are difficult to obtain.

Keywords : Supersolidity, boundary perturbation approach

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1 Introduction

A supersolid (SS) is a solid that possesses an element of superfluidity¹. The existence of SS ⁴He was first suggested 67 years ago², while possible underlying mechanisms were discussed 30 years later³⁻⁴. The subsequent crucial step is the detection of solid ⁴He acquiring nonclassical rotational inertia (NCRI)⁵. NCRI is indicated by a drop in the resonant period of the oscillator below the transition temperature. Recently the experimental evidence for SS was reported by Kim and Chan in a series of torsional oscillator experiments on bulk solid helium⁶ where it was demonstrated that the decrement in the period is proportional to the SS fraction in the limit of low oscillation speed.

Before the discovery of SS, there already were many researches⁷⁻¹² about its essence and existence. However, even after Kim and Chan announced the experimental evidence of SS, Day *et al.* conducted similar experiments¹³ and showed no indication of a mass redistribution in the Vycor that could mimic SS decoupling and put an upper limit of about $0.003 \mu\text{m/s}$ on any pressure-induced SS flow in the pores of Vycor. Later on Day and Beamish's measurements¹⁴ showed no indication of low temperature flow, placing stringent restrictions on SS flow in response to a pressure difference. The average flow speed at low temperatures was less than $1.2 \times 10^{-14} \text{ m/s}$, corresponding to a SS velocity at least 7 orders of magnitude smaller than the critical velocities inferred from the torsional oscillator measurements¹⁵⁻¹⁶. Thus it was claimed that the origin or exact mechanism of free-flowing SS although confirmed but not yet resolved¹⁷⁻²².

Meanwhile, as evidenced by recent experiment (by Grigorev *et al.*)²³ where the temperature dependence of pressure in solid ^4He grown by the capillary blocking technique was measured; at temperatures below 0.3K (where the supersolidity was observed) they found the glassy $\propto T^2$ contribution to pressure. It was then proposed by Andreev²⁴ and Rittner and Reppy²⁵ the observed supersolidity might be in a glassy solid (helium) state. Both idea could also be inspired from the annealing effect of supersolidity reported before¹⁹.

In this short paper, we offer an explanation for the argues between Day *et al.*^{13–14} and Kim and Chan^{15–16} by treating the nanopores which were produced in porous Vycor¹⁵ or porous gold samples¹⁶ to be a kind of (cylindrical) wavy-rough nanotubes. The transport of glassy SS helium²¹ or quantum glass²⁴ inside nanopores which have presumed wavy-rough wall will be investigated here. We adopt the verified model initiated by Cagle and Eyring²⁵ which was used to study the annealing of glass. To obtain the law of annealing of glass for explaining the too rapid annealing at the earliest time, because the relaxation at the beginning was steeper than could be explained by the bimolecular law, Cagle and Eyring tried a hyperbolic sine law between the shear (strain) rate : and (large) shear stress : τ and obtained the close agreement with experimental data. This model has sound physical foundation from the thermal activation process²⁶ (Eyring²⁶ already considered a kind of (quantum) tunneling which relates to the matter rearranging by surmounting a potential energy barrier) and thus it might also resolve the concern raised by Anderson²² for the thermal noises to the superflow of vortex liquid. With this model we can associate the (glassy) fluid with the momentum transfer between neighboring atomic clusters on the microscopic scale and reveals the atomic interaction in the relaxation of flow with dissipation (the momentum transfer depends on the activation shear volume, which is associated with the center distance between atoms and is proportional to RT/τ_0 (R is the gas constant, T temperature in Kelvin, and τ_0 a constant with the dimension of stress)).

To consider the more realistic but complicated boundary conditions in the wall of nanotubes, however, we will use the boundary perturbation technique²⁷ to handle the presumed wavy-roughness along the wall of nanotubes. The relevant boundary conditions along the wavy-rough surface will be prescribed below. The contents are organized into three parts : Introduction is firstly presented and then we describe our physical assumptions and the mathematical model. After that we illustrate our results and possible comparison with previous rather scattered measurements. In fact, the qualitative agreement is rather good!

2 Physical and Mathematical Formulations

We shall consider a steady transport of the glassy SS helium in a wavy-rough nanotube of a (mean-averaged radius) and the outer wall being a fixed wavy-rough surface, $r = a + \epsilon \sin(k\theta)$, where ϵ is the amplitude of the (wavy) roughness, and the wave number : $k = 2\pi/L$. Firstly,

this fluid^{25–26} can be expressed as $\dot{\gamma} = \dot{\gamma}_0 \sinh(\tau/\tau_0)$, where $\dot{\gamma}$ is the shear rate, τ is the shear stress, and $\dot{\gamma}_0$ is a function of temperature with the dimension of the shear rate. In fact, the force balance gives the shear stress at a radius r as $\tau = -(r dp/dz)/2$. dp/dz is the pressure gradient along the flow (or tube-axis : z -axis) direction.

Introducing $\chi = -(a/2\tau_0)dp/dz$ then we have $\dot{\gamma} = \dot{\gamma}_0 \sinh(\chi r/a)$. As $\dot{\gamma} = -du/dr$ (u is the velocity of the fluid flow in the longitudinal (z -)direction of the nanotube), after integration, we obtain

$$u = u_s + \frac{\dot{\gamma}_0 a}{\chi} [\cosh \chi - \cosh(\frac{\chi r}{a})], \quad (1)$$

here, u_s is the velocity over the surface of the nanotube, which is determined by the boundary condition. We noticed that Thompson and Troian²⁸ proposed a general boundary condition for transport over a solid surface as

$$\Delta u = L_s^0 \dot{\gamma} (1 - \frac{\dot{\gamma}}{\dot{\gamma}_c})^{-1/2}, \quad (2)$$

where Δu is the velocity jump over the solid surface, L_s^0 is a constant slip length, $\dot{\gamma}_c$ is the critical shear rate at which the slip length diverges. The value of $\dot{\gamma}_c$ is a function of the corrugation of interfacial energy.

With the boundary condition from Thompson and Troian²⁸, we can derive the velocity field and volume flow rate along the wavy-rough nanotube below using the verified boundary perturbation technique²⁷. The wavy boundary is prescribed as $r = a + \epsilon \sin(k\theta)$ and the presumed steady transport is along the z -direction (nanotube-axis direction).

Along the boundary, we have $\dot{\gamma} = (du)/(dn)|_{\text{on surface}}$. Here, n means the normal. Let u be expanded in ϵ : $u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \dots$, and on the boundary, we expand $u(r_0 + \epsilon dr, \theta (= \theta_0))$ into

$$\begin{aligned} u(r, \theta)|_{(r_0 + \epsilon dr, \theta_0)} &= u(r_0, \theta) + \epsilon [dr u_r(r_0, \theta)] + \epsilon^2 [\frac{dr^2}{2} u_{rr}(r_0, \theta)] + \dots = \\ & \{u_{slip} + \frac{\dot{\gamma} a}{\chi} [\cosh \chi - \cosh(\frac{\chi r}{a})]\}|_{\text{on surface}}, \end{aligned} \quad (3)$$

where

$$u_{slip}|_{\text{on surface}} = L_S^0 \dot{\gamma} [(1 - \frac{\dot{\gamma}}{\dot{\gamma}_c})^{-1/2}]|_{\text{on surface}}, \quad u_{slip_0} = L_S^0 \dot{\gamma}_0 [\sinh \chi (1 - \frac{\sinh \chi}{\dot{\gamma}_c / \dot{\gamma}_0})^{-1/2}]. \quad (4)$$

Now, on the outer wall²⁷

$$\begin{aligned} \frac{du}{dn} &= \nabla u \frac{\nabla(r - a - \epsilon \sin(k\theta))}{|\nabla(r - a - \epsilon \sin(k\theta))|} = [1 + \epsilon^2 \frac{k^2}{r^2} \cos^2(k\theta)]^{-\frac{1}{2}} [u_r|_{(a + \epsilon dr, \theta)} - \\ & \epsilon \frac{k}{r^2} \cos(k\theta) u_\theta|_{(a + \epsilon dr, \theta)}] = u_{0r}|_a + \epsilon [u_{1r}|_a + u_{0_{rr}}|_a \sin(k\theta) - \\ & \frac{k}{r^2} u_{0_\theta}|_a \cos(k\theta)] + \epsilon^2 [-\frac{1}{2} \frac{k^2}{r^2} \cos^2(k\theta) u_{0r}|_a + u_{2r}|_a + u_{1_{rr}}|_a \sin(k\theta) + \end{aligned}$$

$$\frac{1}{2}u_{0,rrr}|_a \sin^2(k\theta) - \frac{k}{r^2} \cos(k\theta)(u_{1,\theta}|_a + u_{0,\theta r}|_a \sin(k\theta)) + O(\epsilon^3). \quad (5)$$

Considering $L_s^0 \sim a \gg \epsilon$ case, we also presume $\sinh \chi \ll \dot{\gamma}_c/\dot{\gamma}_0$. With equations (1) and (5), using the definition of $\dot{\gamma}$, we can derive the velocity field (u) up to the second order. The key point is to firstly obtain the slip velocity along the boundary or surface. After lengthy mathematical manipulations, we obtain the velocity fields (up to the second order) and then we can integrate them with respect to the cross-section to get the volume flow rate (Q , also up to the second order here) :

$$Q = \int_0^{\theta_p} \int_0^{a+\epsilon \sin(k\theta)} u(r, \theta) r dr d\theta = Q_{smooth} + \epsilon Q_{p0} + \epsilon^2 Q_{p2}. \quad (6)$$

Here, θ_p could be 2π or $2\pi/k$ depending on the specific requirement for dimensionless consideration.

3 Results and Discussions

We firstly compare the velocity fields between the smooth and wavy-rough nanotubes in Fig. 1 for the mean radius $a = 3.5$ nm (cf. Refs. 15 and 20). The wave number k is fixed to be 10. We select the amplitude of wavy-roughness ϵ to be 0.02 and 0.04 a (to check the geometry effect which is valid for small ϵ here due to the perturbation approach). We try $\dot{\gamma}_0 = 10000 \text{ s}^{-1}$, $\dot{\gamma}_0/\dot{\gamma}_c = 0.1$ (cf. Ref. 29). We then set $L_s^0 = 0.5a$. The x-axis (χ/a) shown in Figure 1 is the ratio of forcing (along the z -axis direction) per unit volume and (shear) stress. The y-axis is for the total velocity (u), up to the second order. Note that the real range of the (referenced) shear rate : $\dot{\gamma}_0$ will depend on the specific material chosen as well as the experimental procedure²⁹. We can observe the wavy-roughness (ϵ) will significantly enhance the flow rate³⁰ due to larger surface-to-volume ratio and *slip* effect (along the wall)³¹. There is net flow rate even χ/a is zero (without forcing!)³⁰ (it could be a persistent current or a spontaneous flow without a pressure difference and the velocity of this supeflow is proportional to the small amplitude of wavy-roughness, the (referenced) shear rate and the slip length). Our results agree with those of Ref. 21 (measured higher SS fraction) for larger disordered or glassy helium cases (having much larger roughness than that of Refs. 15 and 16).

Once $\chi \sim 0.1$, the velocity for the smooth nanopore will be around 10 $\mu\text{m/s}$ which was reported in Ref. 15. The upper limit of u set in Ref. 13 by Day *et al.* could be due to χ less than 0.0001 (presumed their sample of smooth nanopores, too)! We remind the readers that the geometry effect for the enhanced flow rate (for small wavy-roughness and smaller forcing) is clear as shown in this figure (once ϵ is increasing) but we should keep in mind that the perturbation approach might limit ϵ to be less than 0.1 a .

To check the larger pore effect ($a = 245$ nm in Ref. 16), we calculate the total velocity for $a=245$ nm case while keeping all other parameters almost the same. The trend (differences between

the smooth and rough nanotubes) shown in Fig. 2 is almost the same as that in Figure 1. We can observe that, under the same selected parameters, for larger smooth nanopores, the velocity should be much larger than that in smaller smooth nanopores (at least 2 order of magnitudes larger). However, as reported in Ref. 16 for porous gold cases, the velocity is almost the same order of magnitude as that in Ref. 15. There might be agreement between our results and those in Ref. 16 for $a = 245$ nm case if $\dot{\gamma}_0 = 10000 \text{ s}^{-1}$ is lowered to less than 100 s^{-1} ! As the nanopore size in Ref. 14 (glass capillaries) is unknown, thus to obtain the rather small velocity ($\leq 1.2 \times 10^{-6} \mu\text{m/s}$) reported therein χ should be less than 10^{-8} . Note that those lowest curves illustrated in Figs. 1 and 2 were obtained using the Navier-slip boundary conditions (e.g., cf. Ref. 32).

In brief summary, we have theoretically obtained the velocity (up to the second order) inside the wavy-rough cylindrical nanopores by using the glassy (solid) helium-fluid model and boundary perturbation method. Our results show that the calculated velocity for smaller (presumed) smooth nanopores (radius ~ 3.5 nm) could be of the same order of magnitude as those in Ref. 15 after carefully selected parameters. Those measurements reported in Ref. 14, however, as the size of nanopores are unknown, are difficult to be reproduced using our approach! We shall investigate more complicated issues³³ in the future.

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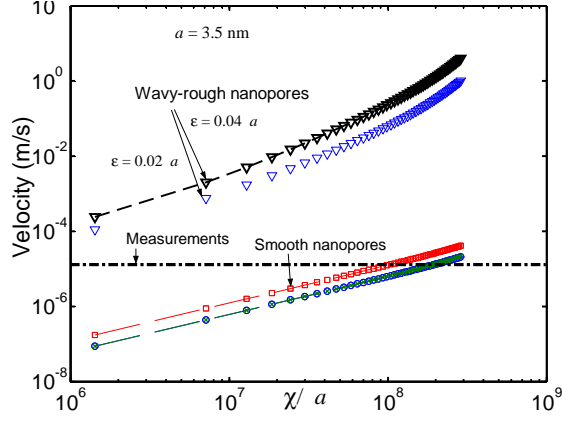


Fig. 1 Calculated velocity fields for different χ/a (forcing (along the z -axis direction) per unit volume and referenced shear stress). $a = 3.5$ nm. ϵ ($=0.02$ and $0.04 a$ here) is the amplitude of wavy roughness. The wave number of roughness (k) is 10 here. Kim and Chan's data¹⁵ could be reproduced once χ is around 0.1 and the slip length (L_s^0) is $0.5 a$. The lowest curves are only due to boundary effects (i.e., boundary slip or contributions from the slip length term in equation (7)) for smooth nanopores. As the experimental data were quite scattered and we only put the roughly averaged value here for comparison.

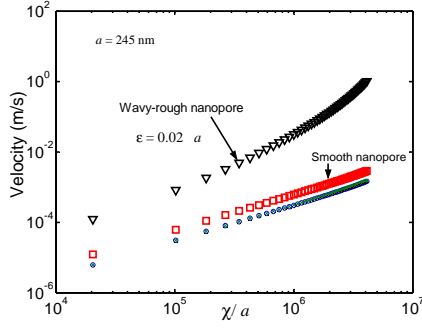


Fig. 2 Calculated velocity fields for different χ/a (forcing (along the z -axis direction) per unit volume and referenced shear stress). $a = 245$ nm. ϵ ($=0.02 a$ here) is the amplitude of wavy roughness. The wave number of roughness (k) is 10 here. Kim and Chan's data¹⁶ could be reproduced once $\dot{\gamma}_0$ (the shear rate) is lowered down to 100 s^{-1} . The lowest curves are only due to boundary effects (i.e., boundary slip or contributions from the slip length term in equation (7)).